

Converges by direct comparison

$$16. \sum_{n=1}^{\infty} \frac{5n^3 - 3n}{\cancel{n^2(n+2)(n^2+5)}} < \sum_{n=1}^{\infty} \frac{5n^3}{n^5} = \sum_{n=1}^{\infty} \frac{5}{n^2}$$

$(n+2)(n^2+5)$
 $n^3 + 2n^2 + 5n + 10$

\uparrow
 p-series
 $p=2$
 converges

or limit comparison

$$\lim_{n \rightarrow \infty} \frac{5n^3 - 3n}{n^5 + \dots} \cdot \frac{n^5}{5n^3} = 1$$

\uparrow
 says: both series
 behave same

Feb 22-11:47 AM

$$17. \sum_{n=1}^{\infty} \frac{3^{n-1} + 1}{3^n} \quad \text{diverges by } n^k \text{ term test}$$

$$\sum_{n=1}^{\infty} \frac{3^{n-1}}{3^n} + \frac{1}{3^n}$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{3} + \frac{1}{3^n} \right) \rightarrow \frac{1}{3}$$

$$n^k \text{ term test} \quad \lim_{n \rightarrow \infty} \frac{1}{3} + \frac{1}{3^n} = \frac{1}{3} \quad \uparrow \text{ not } 0$$

Feb 22-12:00 PM

9.5 Absolute vs Conditional Convergence

intervals of convergence

absolute convergence - $\sum |a_n|$ converges

or ~~and~~ $\sum a_n$ converges
then

conditional convergence - $\sum |a_n|$ diverges

or but $\sum a_n$ converges

neither: both diverge
↑ has some neg terms
use AST

Feb 22-12:05 PM

does $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ converge absolutely, conditionally or neither.

seq: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$

start with abs. $\sum_{n=1}^{\infty} \left| (-1)^{n+1} \frac{1}{n} \right| = \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \dots$
series = sum of all terms

without abs

p-series $p=1$
harmonic } diverges

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \dots$$

converges by AST:

1. signs alternate ✓
2. /terms/ get smaller ✓
3. $a_n \rightarrow 0$ ✓

Feb 22-12:12 PM

What does $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \dots$ converge to?

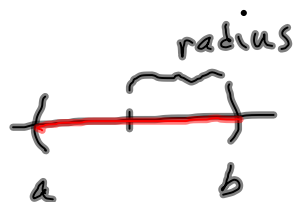
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \dots (-1)^{n+1} \frac{x^n}{n}$$

let $x=1$

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \dots$$

Feb 22-12:25 PM

Intervals of convergence - values of x for which the series converges



1. do the interior (ratio test)

2. then do endpoints

Feb 22-12:30 PM

Ex 6 a $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{2n}$

find the interval
of convergence

1. ratio test

$$\lim_{n \rightarrow \infty} \left| \frac{x^{2(n+1)}}{2(n+1)} \cdot \frac{2n}{x^{2n}} \right| = \lim_{n \rightarrow \infty} \left| x^2 \cdot \frac{2n}{2n+2} \right|$$

$\frac{x^{2n+2}}{x^{2n}} = x^2$

$$= x^2 < 1$$

$-1 \leq x \leq 1$

2. endpoints

$x=1$ plug in $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n} = \frac{1}{2} - \frac{1}{4} + \frac{1}{6} \dots$

$x=-1$ $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(-1)^{2n}}{2n} = \frac{1}{2} - \frac{1}{4} + \frac{1}{6} \dots$

converges by AST

converges by AST

Feb 22-12:35 PM